1. (von Neumann's coin trick)
   Suppose you are given a coin for which the probability of HEADS, say p, is unknown. How can you use this coin to generate unbiased (i.e., $\Pr HEADS = \Pr TAILS = \frac{1}{2}$) coin flips? Give a scheme for which the expected number of flips of the biased coin for extracting one unbiased coin-flip is no more than $\frac{1}{p(1-p)}$.

2. Consider adapting the min-cut algorithm of Section 1.1 to the problem of finding an *s-t min-cut* in an undirected graph. In this problem, we are given an undirected graph G together with two distinguished vertices *s* and *t*. An *s-t* cut is a set of edges whose removal from G disconnects *s* from *t*; we seek an *s-t* cut of minimum cardinality. As the algorithm proceeds, the vertex *s* may get amalgamated into a new vertex as a result of an edge being contracted; we call this vertex the *s*-vertex (initially the *s*-vertex is *s* itself). Similarly, we have a *t*-vertex. As we run the contraction algorithm, we ensure that we never contract an edge between the *s*-vertex and the *t*-vertex.

   Show that there are graphs in which the probability that this algorithm finds an *s-t* min-cut is exponentially small.